

2006年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2006

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数 学 (A)

MATHEMATICS (A)

注意 ☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS (A)

Nationality		No.	
Name	(Please print full name, underlining family name)		
			Marks

1 Fill in the blanks with the correct numbers.

- (1) The solution of the inequality $4x - 6 < 2x < 5x + 3$ is

$$\boxed{\textcircled{1}} < x < \boxed{\textcircled{2}}.$$

- (2) Assume that $x - 2$ is a factor of the polynomial $f(x) = x^3 + ax^2 + bx + 2$ and that $f(x)$ gives a remainder of -3 when it is divided by $x + 1$. Then

$$a = \boxed{\textcircled{1}} \text{ and } b = \boxed{\textcircled{2}}.$$

- (3) The graph of the function $y = x^2 + ax + 1$ touches to the x -axis if and only if

$$a = \boxed{\textcircled{1}} \text{ or } \boxed{\textcircled{2}}.$$

(4) $(\log_2 3)(\log_3 4) + 3^{\log_3 5} = \boxed{}.$

(5) When $\tan A = \sqrt{2}$, then $\frac{\sin A + \cos A}{\sin A - \cos A} = \boxed{}.$

2 The circles $C : x^2 + y^2 + kx + (1+k)y - (k+1) = 0$ pass through the same two points for every real number k .

(1) Find the coordinates of these two points.

(2) Find the minimum value of the radius of a circle C .

3 Take points $A\left(\frac{7}{2}, 0\right)$, $B(0, 7)$ and $C\left(-\frac{7}{6}, 0\right)$ on the xy -plane. The parabola $y = -x^2 + ax + b$ is tangent to both lines BA and BC .

(1) Determine a and b .

(2) Calculate the area of the domain bounded by the line BA , the parabola and the y -axis.