2006年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2006

学科試験　　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数　　学（B）

MATHEMATICS (B)

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS **60 MINUTES.**

MATHEMATICS (B)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Fill in the blanks with the correct numbers.

(1) The solution of the inequality $|2x-1| < x+2$ is

① $\square$ $< x <$ ② $\square$.

(2) The x-axis is tangent to the graph of the function $y = x^2 + ax + 1$

if and only if $a =$ ① $\square$ or ② $\square$.

(3) The minimum of the function $f(x) = (\log_2 x)^2 + \log_4 x + 1$ is $\square$.

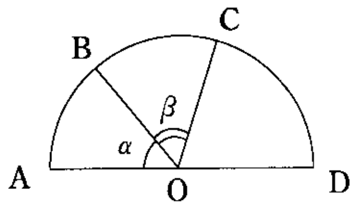
(4) The three points (1, 2, 4), (2, 5, 6), and (① $\square$, ② $\square$, 10) are on the same line.

(5) $\int_0^{\frac{\pi}{2}} x \sin x dx =$ $\square$.

2 Four points A, B, C and D lie on a circle in that order. The radius of this circle is 1 and the center is O. Suppose the line AD is a diameter of this circle and the ratio of the areas of the triangles is

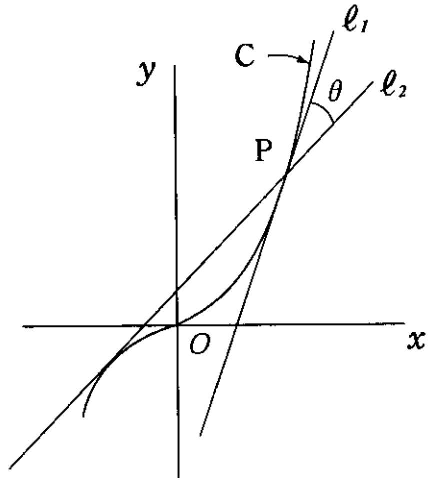
$$\triangle OAB : \triangle OBC : \triangle OCD = 1 : 2 : 2.$$

(1) Let $\alpha = \angle AOB$ and $\beta = \angle BOC$. Find $\sin\alpha : \sin\beta$.



(2) Find the area of the rectangle $ABCD$.

3 Let p be a positive number. Let C be the curve $y = 2x^3$ and $P\ (p,\ 2p^3)$ a point on C. Let l_1 be the tangent line at P and l_2 be another tangent line of C which passes through P.



(1) Express the slope of l_2 in terms of p.

(2) Find $\tan\theta$, where θ is the angle formed by l_1 and l_2 and $0 < \theta < \frac{\pi}{2}$.

(3) Find the maximum value of $\tan\theta$.