2006年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2006

学科試験　　問題

EXAMINATION QUESTIONS

（高等専門学校留学生）
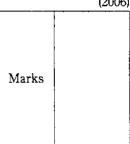
COLLEGE OF TECHNOLOGY STUDENTS

数　　学

MATHEMATICS

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS **60 MINUTES.**

MATHEMATICS

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks

1 Fill in the blanks with correct numbers or expressions.

1) Solve the inequality $(x-1)^2 < x-1$.

①

2) Solve the equation $2^{2x-1}+2 \cdot 2^x - 6 = 0$.

$x =$ ②

3) Solve the equation $\log_2(4-x) - \log_4(x-1) = 1$.

$x =$ ③

4) Solve the inequality $(2\sin x - \sqrt{3})(2\sin x - 1) < 0, (0 \le x \le 2\pi)$.
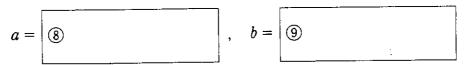
④

5) Solve the inequality $\dfrac{1}{x-1} > \dfrac{1}{x+1}$.

⑤

6) Suppose $z = \dfrac{\sqrt{3}+3i}{\sqrt{3}+i}$, calculate the absolute value r of z and the argument θ $(0° \le \theta < 360°)$ of z. $(i^2 = -1)$

$r =$ ⑥ , $\theta =$ ⑦

7) Let $f(x) = ax^2 - 4ax + b$, $(a > 0)$ be defined in $1 \leqq x \leqq 5$.

Suppose the average of the maximum value and minimum value of the function is 14, and the difference between the maximum value and minimum value is 18. Find the values of a $(a > 0)$ and b.

$$a = \boxed{⑧ \qquad\qquad} , \quad b = \boxed{⑨ \qquad\qquad}$$

8) Find $\displaystyle\lim_{x \to 1} \frac{x-1}{\sqrt{x+8}-3} = \boxed{⑩ \qquad\qquad}$.

9) Find $\displaystyle\int_0^{\sqrt{3}} 3x\sqrt{x^2+1}\,dx = \boxed{⑪ \qquad\qquad}$.

10) Find the value of $x^2 + 3x - 1$ when $x = \dfrac{-3+\sqrt{13}}{2}$.

$$x^2 + 3x - 1 = \boxed{⑫ \qquad\qquad}$$

11) A box contains 3 red balls, 2 blue balls and 5 green balls. Two balls are drawn from the box, one after the other, and not replaced.

What is the probability that they are both blue?

$$\boxed{⑬ \qquad\qquad}$$

What is the probability that they are both the same colour?

$$\boxed{⑭ \qquad\qquad}$$

2 Let $A = \begin{pmatrix} \frac{1}{2} & 0 \\ a & a \end{pmatrix}$.

1) Write A^3 as a function of a.

2) Write A^n as a function of a and n, where n is a positive integer.

3 Suppose $f(x) = x^3 + ax^2 + bx + c$ satisfies $f(-2) = -10$, and takes the extreme value $\frac{50}{27}$ when $x = \frac{2}{3}$. Find the values of a, b and c.

4 Let $C_1 : y = x - \dfrac{1}{2}x^2$ and $C_2 : x = y - \dfrac{1}{2}y^2$ be curves on the xy plane.

1) Find the equation of the tangent to the curve C_1 at $x = k$.

2) Suppose the line obtained in 1) is also tangent to the curve C_2.

Find all values of k and the equations of the tangents.

3) Evaluate the area of the figure enclosed by all tangents obtained in 2) and

the curve C_2.